Takagi-Sugeno Fuzzy Control of Batch Polymerization Reactors

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Abstract — It is well-known fact that batch processes are gaining wider ground in chemical industries. Compared with continuous processes the control of batch processes is more difficult because physical and chemical properties of the contents, such as heat capacity, heat transfer coefficient and reaction rate vary from run to run and within runs.

The control problem focuses on the temperature control of a polystyrene batch reactor with rule based Takagi - Sugeno fuzzy controller with Controller Output Error Method. The proposed learning fuzzy logic controllers are shown to be capable of providing good overall system performance.

I. INTRODUCTION

It is a well-known fact that batch processes are gaining wider ground in chemical industries. The course is motivated first of all by the extraordinary flexibility of batch processes which allow quick adaptation to market demand and are made possible by rapid development of process control. Compared to continuos processes the control of processes are more difficult (complex sequential control task, operating conditions change with time, etc.).

The typical unit of batch systems in polymer, pharmaceutical and fine chemical industries is an autoclave with heating-cooling system, which uses directly cold and hot utility fluids available in the plant given temperatures.

However, the many complications, non-linearities and constrains exist to transform the control problem into a quite challenging one. A further difficulty arises from the fact that polymerization of vinyl monomers is an auto-catalytic reaction. The problem becomes difficult to formulate succinctly and does not lend itself to mathematical formalism. Data and equations are provided for process hardware, reaction kinetics, heat transfer and reaction viscosity[1].

Several researches have addressed control problems in reactor systems of this type. Juba and Hamer [2], and Berber [3] provide an excellent overview on the challenges in batch reactor control and suggest several control strategies.

Davidson [4] proposes a control algorithm which combines knowledge of the process with the logic used by the skilled operator and compares the result with standard PI controllers.

Fuzzy control, a non-mathematical control algorithm based on the set of decision rules has found applications in some industrial areas. Along with the practical successes observed with this control strategy in industry, there has also been some academic interest in fuzzy logic control. As a result, limitations of classical fuzzy control are extended, and a model based approach has been suggested [5].

Examples for applications of fuzzy control to chemical reactors can be found in the literature [6].

Most of the fuzzy controllers developed to date have been of *rule-based* type [11], where the rules in the controller attempt to model the operator's response to particular process situations.

An alternative approach uses fuzzy model in process control (*model-based* fuzzy control).

The aim of our work is to develop control algorithms to control batch polymerization reactors that belong to both of these classes.

In this paper we have applied a rule based Takagi-Sugeno proportional-derivative fuzzy controller algorithm with gradient-descent adaptation (COEM method [19]), and compared with previously developed rule based PI type fuzzy and fuzzy supervised PI controllers [21]

The paper is organized as follows: The control problem is described in section II. In section III. the Takagi-Sugeno fuzzy model, and the learning PD type controller is considered. The simulation methods and results are presented in section IV.

II. THE CONTROL PROBLEM

A. Process Description

A stirred tank reactor is used to produce expanded polystyrene in batch processes.

Most processes consist of the following steps. An initial charge of prepolymer, surfactants, initiators, and monomer mixture is added to the reactor. After this initial stage the reaction mixture is heated to the temperature of the polymerization. The third stage called the impregnation stage, in which the blowing agent, commonly n-pentane is loaded to the reactor. Depending on the impregnation temperature and pressure minimum time must be allowed for diffusion of the blowing agent to reach the core of the beads and for complete monomer exhaustion [9]. The last step is the cooling down of the reaction mixture to the temperature required for further processing.

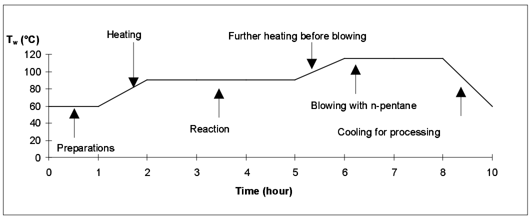

Fig. 1: The prescribed temperature profile

B. Control Configuration

The scheme of the reactor system is summarized in Fig. 2. The reactor temperature is maintained at a desired value by adjusting the temperature for water recirculating through the reactor jacket.

When the controller output is between 0 and 50%, the controller is in cooling mode. In cooling mode, the controller output is used to control the Sv valve.

For controller outputs between 50 and 100% the controller is in heating mode. The controller is used to throttle a control valve referred to as the steam valve (Sg). While the steam valve is open, medium pressure steam is injected directly into the recirculating water steam, thereby adding heat.

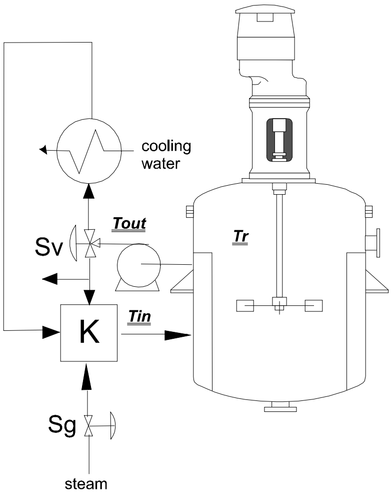

Fig. 2: Scheme of the reactor system.

III. STRUCTURE OF FUZZY CONTROLLERS

A. Takagi and Sugeno's Fuzzy Model

In this paper we deal with Takagi and Sugeno's fuzzy model [8,10]. This fuzzy model can be formulated as the following form:

$$L^i: \quad IF\ x(1)\ is\ A_1^i\ and\ \ldots\ and\ x(n)\ is\ A_n^i\ THEN \quad y^i = a_0^i + a_1^i \cdot x(1) + \ldots + a_n^i \cdot x(n) \tag{1}$$

where $L^i (i = 1,2,\ldots l)$ denotes the i-th implication, l is the number of fuzzy implications, y^i is the output from the i-th implication, $a_p^i (p = 0,1,\ldots n)$ are consequent parameters, $x(1),\ldots,x(n)$ are the input variables, and A_p^i are fuzzy sets whose membership functions are denoted by the same symbols as the fuzzy values.

Given an input $(x(1),\ldots.x(n))$, the final output of the fuzzy model is inferred by taking the weighted average of the y^i's:

$$y = \frac{\sum_{i=1}^{l} w^i y^i}{\sum_{i=1}^{l} w^i} \tag{2}$$

where $w^i > 0$, and y^i is calculated for the input by consequent equation of the i-th implications, and the weight w^i implies the overall truth value of premise of the i-th implication for input calculated as

$$w^i = \prod_{p=1}^{n} A_p^i\left(x(p)\right) \tag{3}$$

B. PD -like Takagi-Sugeno Fuzzy Logic Controller (FLC)

Several examples of realizations of PID controllers by fuzzy control methods can be found in the literature [7,11,12,13,14].

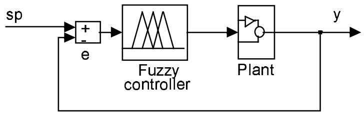

Fig. 3: Scheme of the control loop (realized in MATLAB®/Simulink)

From the PD controller form

$$u(t) = K_p^c\left(e(t) + K_D^C \cdot \frac{de(t)}{dt}\right) + bias \tag{4}$$

with discretisation we obtain

$$u_{PD}(kT) = K_p e(kT) + K_{PD} \cdot \Delta e(kT) \tag{5}$$

where $K_p = K_p^c \qquad K_I = \dfrac{K_p^c \cdot K_I^c}{T}$. (6)

where T is the sampling time.

From equation (1) and (6) the set of rules of the FLC can be expressed with the following form:

$$\begin{aligned} L^i: \quad & IF\ e(kT)\ is\ A_1^i\ and\ \Delta e(kT)\ is\ A_2^i\ and \\ & x(1)\ is\ B_1^i\ and\ \ldots\ and\ x(n)\ is\ B_n^i \\ & THEN \quad u(kT)^i = a_1^i e(kT) + a_2^i \Delta e(kT) \end{aligned} \tag{7}$$

where $u(kT)^i$ denotes the output from the i-th implication, $a_p^i (p = 1,2)$ are consequent parameters of the sub-PD controller, $x(1),\ldots.x(n)$ are the auxiliary input variables (not necessary), A_p^i and B_p^i are fuzzy sets on $e(kT)$ and $\Delta e(kT)$ and on auxiliary input variables, whose membership functions are denoted by the same symbols as the fuzzy values.

The final output of the fuzzy controller is inferred by taking the weighted average of the u^i's:

$$u(kT) = \frac{\sum_{i=1}^{l} w(kT)^i u(kT)^i}{\sum_{i=1}^{l} w(kT)^i} \tag{8}$$

where $w^i > 0$, and $u(kT)^i$ is calculated from the input with consequent equation of the i-th implication, and the weight w^i implies the overall truth value of premise of the i-th implication for input calculated as

$$w(kT)^i = A_1^i\left(e(kT)\right) \times A_2^i\left(\Delta e(kT)\right) \times \prod_{p=1}^{n} B_p^i\left(x(p)\right) \tag{9}$$

C. Controller Output Error Method

The Controller Output Error Method (COEM) can be used for on-line tuning of a fuzzy controller [19]. This method can be used with any fuzzy controller design, the only requirement being that the controller is capable of stabilizing the plant before the commencement of tuning.

COEM does not perform a system identification and does not require the plant output error to be propagated backwards to the plant input through a reference model, as in direct adaptive control, or directly through the plant as in.

The controller output error is an error by $e_u = u(kT) - \hat{u}(kT)$. It is important to note that, $\hat{u}(kT)$ is calculated by the controller, but it is not applied to the plant.

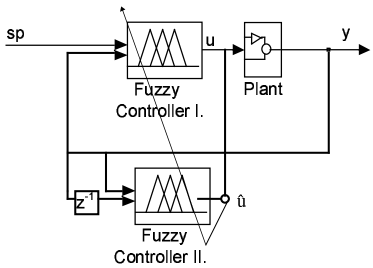

Fig. 4: Scheme of the COEM control loop

$\hat{u}(kT)$ is calculated by producing a new controller input vector $\hat{z}(kT)$, which is passed through the fuzzy controller as follows:

$$\hat{u}(kT) = f\left(\hat{z}(kT), \theta(k,T)\right) \tag{10}$$

where:

$$\hat{z}(kT) = \begin{bmatrix} y(kT+T), y(kT), \ldots, y(kT-nT), \\ u(kT-T), \ldots, u(kT-mT) \end{bmatrix} \tag{11}$$

where $\theta(kT)$ is the parameter vector of the controller.

The input vector $\hat{z}(kT)$ only differs from $z(kT)$ in the first element, where $y(kT+T)$ replaces $sp(kT+T)$ (set point) is passed to the plant. Note also that, for each time instance, two control commands $u(kT)$ and $\hat{u}(kT)$ are produced, although only one of these $u(kT)$, is passed to the plant.

The controller output error $e_u(kT)$, is used in a cost function such as the following

$$J(kT) = \frac{1}{2}\left(e_u(kT)\right)^2 \tag{12}$$

The aim of the adaptation of the controller parameters $\theta(kT)$ is to minimize this cost function with gradient-descent method by the following rule:

$$\theta(kT+T) = \theta(kT) - \eta \frac{\partial J(kT)}{\partial \theta(kT)} \tag{13}$$

where $\eta > 0$ is the learning rate.

In our study we used partial updating, where only the parameters of the consequent part of the rules are updated.

IV. SIMULATION RESULTS

A. Developing the Control Algorithms

In order to compare the control algorithms developed the following performance index was defined:

$$Q = \sum_{i=1}^{N} e_i^2 + \lambda \sum_{i=1}^{N} \Delta u_i^2 \tag{14}$$

where:

$e_i = sp_i - y_i$	the plan output error in the i-th discrete time step,
$\Delta u_i = u_i - u_{i-1}$	the change of the control signal in the i-th time step,
$N = \frac{t_{\max}}{T}$	the maximal number of time steps,
T	is the sampling time
λ	is the weighting factor to maintain the dynamics of the control signal (0.2).

The simulator was built using MATLAB® and C based on a priori mathematical models according to [9,16,17,18].

B. Conventional solution

Among the conventional solutions the PD controller has been examined. The parameters of the controller were determined by optimization with Sequential Quadratic Programming method with MATLAB® *Optimization Toolbox's* *constr* function [20].

The performance index (Q) was 2.11.

C. Applying PD-controller with COEM

In [15] it has been showed that in learning closed loop control system incorporating an integral term tends to destabilize the system. This term is thus not used by using this algorithm, so the rules of the controller:

$$L^i: \quad T(kT) \text{ is } A_1^i \text{ THEN}$$
$$u(kT)^i = a_1^i e(kT) + a_2^i \Delta e(kT) \tag{15}$$

where *T(kT)* is the temperature in the reactor.

We designed triangular membership functions as illustrated in Fig. 5.

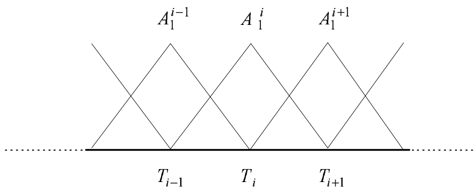

Fig. 5: Membership functions of A_1^i on reactor temperature

The $\theta(kT)$ parameter set ($\left[\underline{a}_1(kT),\ \underline{a}_2(kT)\right]$) was adapted by equation (12), where the partial derivative of cost function $J(kT)$ with respect to each parameters is given below:

$$\frac{\partial J(kT)}{\partial a_1^i(kT)} = -\frac{e_u(kT)\cdot e(kT)}{\sum_{i=1}^{l} w(kT)^i} \qquad (16)$$

$$\frac{\partial J(kT)}{\partial a_2^i(kT)} = -\frac{e_u(kT)\cdot \Delta e(kT)}{\sum_{i=1}^{l} w(kT)^i} \qquad (17)$$

where *l* was set 5.

The performance of this algorithm is shown by Fig. 6., and the performance index (Q) was 1.31.

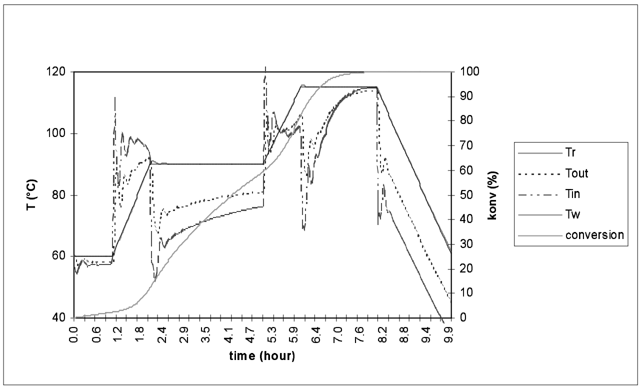

Fig. 6: Performance of the PD -controller with COEM

V. CONCLUSION

In this study the batch polymerization reactors temperature control by Takagi-Sugeno fuzzy controller through the Expanded Polystyrene (EPS) process was investigated.

- In order to solve the problem the simulation program of the technology in MATLAB® Simulink, based on rigorous chemical engineering model, was developed.
- The parameters of the optimal PD controller according to a given cost function were determined .
- We used COEM method to adapt our PD type fuzzy controller.

As the table below (* is presented in [21]) demonstrating the result and the derivation of algorithms shows that we have achieved 5-30% improvement of the performance index characterising the operation of the controller by fuzzy realization of the controllers.

TABLE I
Comparison of various controllers

Control algorithm	*Performance Index*
Optimal PI*	1.883
Optimal gain-varying PI*	1.633
Optimal PD	2.11
Optimal PI type fuzzy*	1.579
Optimal fuzzy supervised PI controller*	1.509
PD type fuzzy controller with COEM method	*1.31*

According to our experiences the conventional control solutions (PID algorithms, PID modifications, adaptation) can be realized by choosing the proper fuzzy structure (non equivalent connection) and can be improved by applying Takagi-Sugeno fuzzy models and Controller Output Error Method.

This type of application can be a proper tool where the already existing linear control algorithm is to be replaced by a better, more efficient nonlinear control algorithm.

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